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An instrument to determine the standard deviation of the counting rate of a series of pulses.

THE UNIVERSITY OF ALBERTA

AN INSTRUMENT TO DETERMINE THE STANDARD DEVIATION OF THE COUNTING RATE OF A SERIES OF PULSES

A DISSERTATION

SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

FACULTY OF ARTS AND SCIENCE

by

Leonard H. Greenberg

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University of Alberta

Faculty of Arts and Science

Department of Physics

The undersigned hereby certify that they have read and recommend to the School of Graduate Studies for acceptance, a thesis entitled "An Instrument to Determine the Standard Deviation of the Counting Rate of a Series of Pulses"
submitted by Leonard H. Greenberg, B.Sc.
in partial fulfilment of the requirements for the degree of Master of Science.

Professor

Professor

Date .... Professor

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Introduction

A radioactive substance of long half life emits particles at a nearly constant average rate, but the number of disintegrations in any time interval will vary because of their random distribution. It has been shown (2,9) that the number will be distributed about the mean value in accordance with the Poisson distribution, and that the standard deviation in the number will be given by the square root of the mean.

If now, counts are made of the number of disintegrations in a given time interval, it may be found that the observations do not follow a Poisson distribution because of counting defects. This may be checked by comparing the standard deviation of the counts obtained with the square root of the mean count. One may also use the Chi square test to determine the probability that the counts obtained are from a random source. Marked effects on the counting rate and on the observed standard deviation will be produced by such counting defects as:

(a) Significantly long dead time (2,3,4)

(b) Two or more pulses being recorded for each one entering the counter.

(c) Pick up of 60 cycle line frequency.

(d) Intermittent counting due to varying voltage on the counter.

To make a reliable measurement of the standard deviation or to apply a Chi square test requires a large number of readings and lengthy calculations. The instrument described in this thesis elimin-

ates these calculations and gives an indication of the counting rate and the standard deviation simultaneously while the counting is being done.

Evaluation of the Counting Rate.

A. Scaler:

The most widely used method of determining counting rate is to make an actual count of the number of pulses received in a measured time interval. An electronic scaler is used to count the pulses. Many types of scalers have been developed (14,15,16,17,18,19).

Some of the advantages and disadvantages of using a scaler may be listed as follows:

Advantages: 1. An actual count is made of each individual pulse so there is no limitation imposed on the accuracy of the result obtained by the accuracy of the electronic components involved.

2. A series of readings over a constant time interval may be used to obtain the standard deviation and then the reliability of the results.

Disadvantages: 1. Especially at high counting rates the accuracy of the results is limited by the precision with which the time interval may be measured. An error in timing should be obtained and considered when estimating the reliability of the results.

2. To obtain an estimate of the reliability of the results, the very long procedure outlined in a later section must be followed.

Interpretation of Scaler Data.

The statistics of counting is thoroughly dealt with in existing literature, ~~such as~~ (1,3,9,21,23). An outline of the method of analysis and interpretation used here follows.

Symbols: - The symbols used in the following section with their definitions are:

M - the mean counting rate

m - the average number of pulses recorded in an interval.

T - the constant interval over which the pulses are counted.

σ_M - the standard deviation of the counting rate. See appendix 1.

σ_m - the standard deviation of the number counted in the time interval T.

x_i - the actual number recorded in the i th interval T.

n - the number of intervals T.

P(x) - the probability of obtaining x counts in the interval T.

τ - the dead time of the counter.

m.d. - mean deviation.

The average number recorded in the interval is given by

$$m = \frac{1}{n} \sum_{i=1}^{n} x_i$$

The mean counting rate is then $\frac{m}{T}$.

The variance in m is obtained by taking

$$\sigma_m^2 = \frac{1}{n} \sum_i (x_i - m)^2 = \frac{1}{n} \sum (x_i^2) - m^2 \qquad (1, p36)$$

The standard deviation is then the square root of the variance.

If n is large so the expected Poisson distribution approaches the normal distribution, the standard error in σ_m is given by (1, p 137) $\sigma_n / \sqrt{2n}$

The fractional standard error is then $1/\sqrt{2n}$

Table 1 shows the standard error as a percentage of the standard deviation for various numbers of samples, n. Fig. 1 is a plot of the standard error in the standard deviation for numbers of samples from 10 to 10,000.

TABLE 1.

Number of Samples n.	% s.e. in σ_m
10	22.4
100	7.1
200	5.0
5, 000	1.0

Chi Square Test: (1,p.173)

The Chi Square test is used to find the probability of obtaining at least as large a standard deviation as that obtained if the samples follow the Poisson distribution.

If S^2 is an unbiased estimate of the variance of n samples, and σ is the expected standard deviation, then $\frac{nS^2}{\sigma^2}$ will be distributed like Chi squared with n-1 degrees of freedom. For a Poisson distribution the expected variance is equal to the mean m. From a series of readings, $nS^2 = \sum_i (x_i - m)^2$

Then $\frac{1}{m} \sum_{x_i} (x_i - m)^2$

will be distributed like Chi Squared

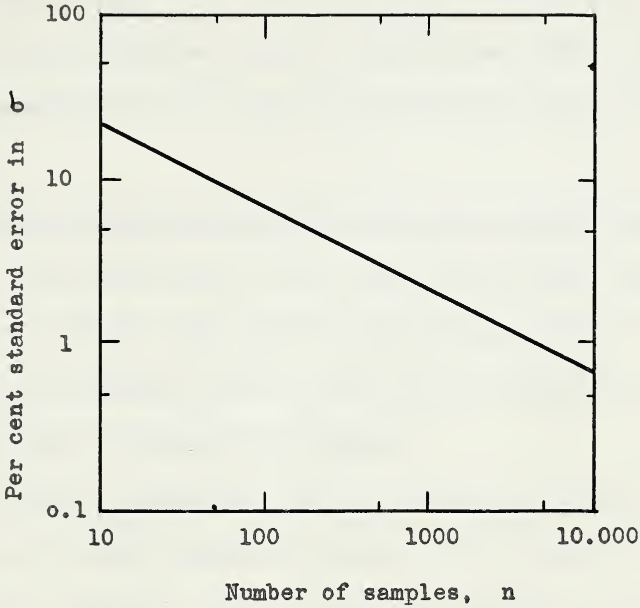


Fig. 1.

The approximate per cent standard error in σ for various numbers of samples. The error is computed from $\frac{s.e}{\sigma} = \frac{1}{\sqrt{2n}}$ which holds only for large n and large m.

with n-1 degrees of freedom.

The χ^2 tables then tell the probability that a value of S greater than the value obtained will be found if the samples are random. Tables of χ^2 are to be found in many books on statistics, (1,p.171). Fig. 2 is a copy of a chart showing the probability of obtaining the given value of χ^2 for n-1 degrees of freedom. The reproduction is from one distributed to classes by R.D.Evans at M.I.T.

Comparison of the Mean Deviation With the Standard Deviation.

In the construction of the meter it was found that the average deviation from the mean could be more easily obtained than the standard deviation. However, the standard deviation is the quantity usually used in statistical analysis.

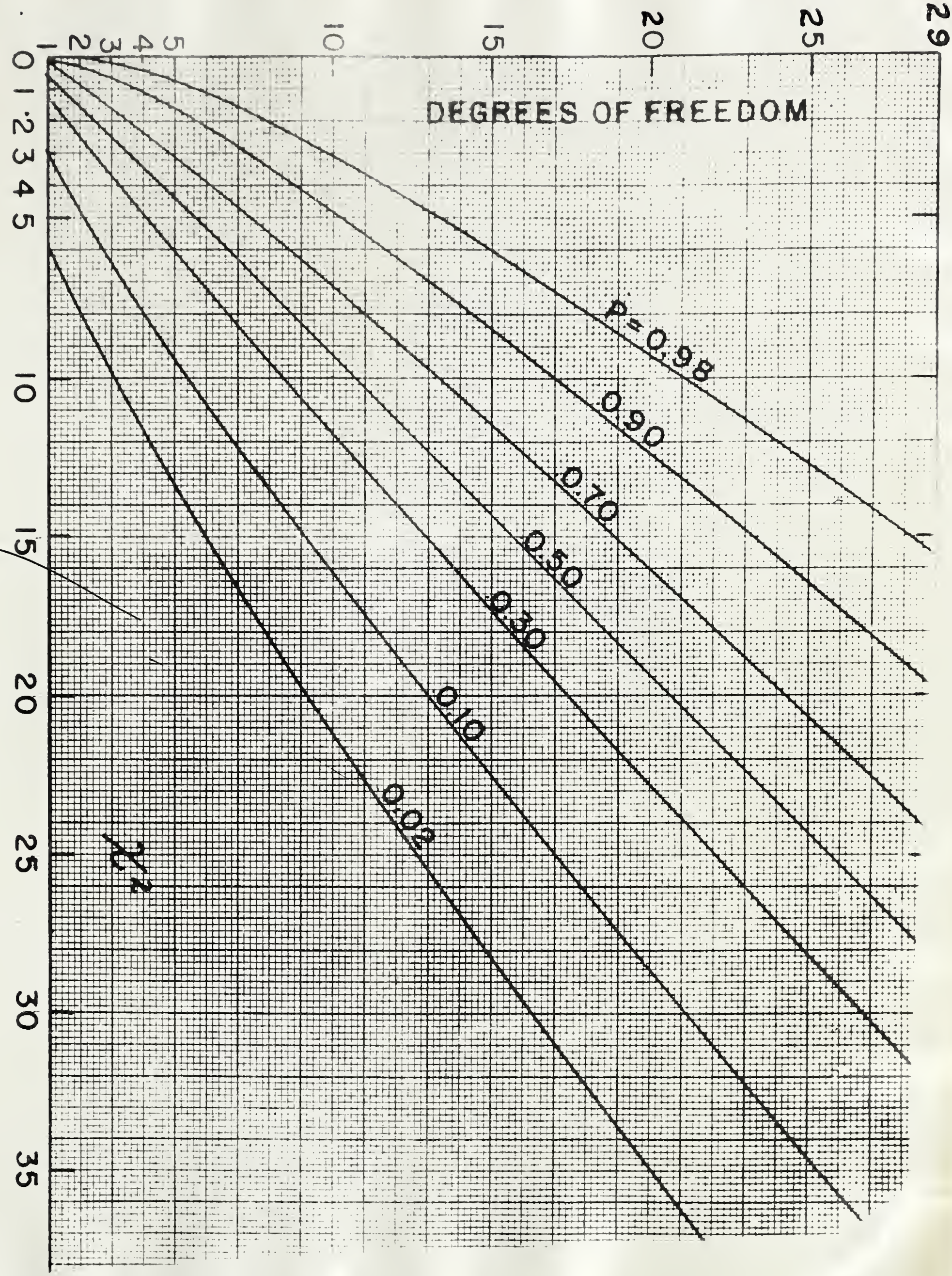
It will be shown that for the Poisson and normal distributions there exist simple relations between the standard and the mean deviations. Because of this it has been possible to calibrate the instrument in terms of standard deviation although it is mean deviation which is measured.

It should be noted that the calibration takes account of Poisson and normal distributions only. For other distributions the indicated standard deviations may not be correct.

For a continuous distribution, for which the probability density is p(x), the variance is given by the second moment about the mean, or

$$\sigma^2 = \int_{-\infty}^{\infty} (x-m)^2 \, p(x) \, dx$$

FIG. 2



The mean deviation is

$$m.d. = \int_{-\infty}^{\infty} |x-m| \; p(x) \; dx$$

For a discrete distribution the variance is given by:

$$\sigma^2 = \sum_i (x_i-m)^2 \; P(x)$$

and the mean deviation is

$$m.d. = \sum_i |x_i-m| \quad P(x)$$

In each case $|x-m|$ refers to the absolute value.

Normal Distribution:

The probability density for the normal distribution is given by (1,p. 55)

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \; e^{\frac{-(x-m)^2}{2\sigma^2}}$$

The variance is σ^2 and the standard deviation is σ.

The mean deviation may be found by evaluating

$$\int_{-\infty}^{m} \frac{(m-x)}{\sigma\sqrt{2\pi}} \; e^{-\frac{(x-m)^2}{2\sigma^2}} dx + \int_{m}^{\infty} \frac{x-m}{\sigma\sqrt{2\pi}} \; e^{-\frac{(x-m)^2}{2\sigma^2}} dx$$

This may be simplified by putting $\frac{(x-m)^2}{2\sigma^2} = u$.

Then $du = \frac{(x-m)dx}{\sigma^2}$, and one has

$$m.d. = -\frac{\sigma}{\sqrt{2\pi}} \int_{\infty}^{0} e^{-u} du + \frac{\sigma}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-u} du$$

$$= \sigma\sqrt{\frac{2}{\pi}} = 0.798\,\sigma$$

Therefore, for a normal distribution the mean deviation is 0.798 σ.

If the mean is large, the Poisson distribution approximates a normal distribution. We may then expect that for m large, the mean deviation of the Poisson distribution is also given by 0.798 σ.

Poisson Distribution:

For a Poisson Distribution the probability of obtaining x counts if the mean is m is

$$P(x) = \frac{m^x e^{-m}}{x!}$$

The variance or second moment about the mean is

$$\sigma^2 = \sum_{x=0}^{\infty} (x-m)^2 P(x).$$

This can be shown to be equal to the mean, m. Therefore, the standard deviation $\sigma = \sqrt{m}$.

The mean deviation for the Poisson distribution will be found from

$$\sum_{x=0}^{[m]} (m-x)\, P(x) + \sum_{[m]+1} (x-m)\, P(x)$$

where $[m]$ refers to the integer equal to or first below m.

This can be shown to give

$$\text{m.d.} = 2 \quad m \quad P_m([m]) \qquad \text{see appendix 2.}$$

The ratio of the mean deviation to the standard deviation is then:

$$\frac{\text{m.d.}}{\sigma} = 2\sqrt{m} \quad P_m([m])$$

This ratio is plotted against the mean in Fig. 3. It shows that the average value for this ratio approaches the value for the Normal distribution and becomes practically independent of m when m gets greater than about 5.

The Poisson distribution allows only integral values of x, that is an integral number of pulses to be recorded in any time interval. In a counting rate meterx refers to the effective number of pulses stored in the tank circuit at any time. This will not be limited to integral values. The Poisson distribution will then not be applicable. It will be expected to give an approximation at least for integral m. For non integral m it is to be expected that the curve will be smooth between the points obtained for the integers. This is shown by the dotted curve in Fig, 3.

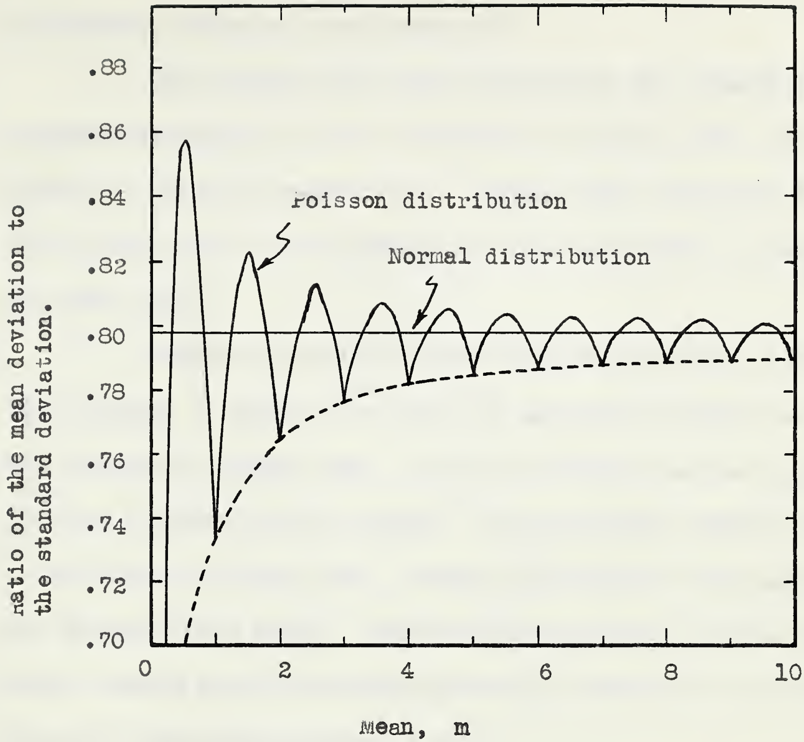


Fig. 3.

The ratio of the mean deviation to the standard deviation for the Poisson and Normal distributions.

Effect of dead time on statistics:

Counter dead times may be classified into two categories, extended or non extended.

Non extended dead times occur when the counter is rendered insensitive to the detection of another pulse for a period τ after it records one. Another pulse occurring in this period will not be counted and will not further extend the dead time.

Extended dead times occur when the dead time is extended for a period τ after the receipt of each pulse whether or not the counter is already dead. Pulses arriving when the counter is dead, however are not counted. Most counters exhibit part of both types of dead times, though G.M.counters have mainly non extended dead times. Scintillation counters, on the other hand, exhibit mostly extended dead times because of the decay time for each scintillation, (10).

G.M. counters have dead times of 300 to 600 micro-seconds, and proportional and scintillation counters are dead for about 0.01 microseconds. Scalers usually have resolving times of from 2 to 10 microseconds. This is insignificant when used with G.M. counters but large when used with scintillation counters. Some of the more recent advances in scaler design have been in the reduction of dead time (20).

W. Feller (2) has made an analysis of the expected number of counts to be recorded, and the standard deviation for both types of dead times. C. Clark (4) has also made an analysis of statistics and counter dead times.

For non extended dead time, Feller shows that the expected number to be counted is

$$\sigma_m^2 \sim \frac{MT}{(1+M\tau)^3} \qquad (2, p.112)$$

and the variance is

$$m \sim \frac{MT}{1+M\tau} + \frac{M^2\tau^2}{2(1+M\tau)^2} \qquad (2, p.112)$$

For an extended dead time he shows that the expected number is

$$m = 1 - e^{-M\tau} + M e^{-M\tau}(T-\tau) \qquad (2, p.115)$$

and the variance in this case will be:

$$\sigma_m^2 = Me^{-M\tau}(T-\tau)\left(1-2M\tau e^{-M\tau}\right) - e^{-M\tau} + (1+M\tau)^2 e^{-2M\tau}$$

The quantities which are measured are m, T, and σ_m^2. If m counts are received and the distribution is Poissonian, the expected variance will be equal to m and the ratio of σ_m^2/m will be one. If there are counting losses due to dead tim , this ratio will decrease as shown in the following analyses.

(a) <u>Non extended dead time</u>

Feller's equations are: as given above.

The ratio may then be written as:

$$\frac{\sigma_m^2}{m} \qquad \frac{2MT(1+M\tau)^2}{(1+M\tau)^3\left\{2MT(1+M\tau)+M^2\tau^2\right\}}$$

neglecting $M^2\tau^2$ with respect to $2MT$ this reduces to

$$\frac{\sigma_m^2}{m} \qquad \frac{1}{(1+M\tau)^2}$$

One of the curves in Fig. 3. shows $\frac{\sigma_m^2}{m}$ plotted as a function of $M\tau$ for non extended dead time.

(b) Extended dead time.

In this case we use the equations showed by Feller (2),

$$m = 1 - e^{-M\tau} + Me^{-M\tau}(T-\tau)$$

and $$\sigma_m^2 = Me^{-M\tau}(T-\tau)(1-2M\tau e^{-M\tau}) - e^{-M\tau} + (1+M\tau)^2 e^{-2M\tau}$$

If T is large compared with τ, then $(T-\tau) \rightarrow T$. Then it will usually follow in counting that $MT \gg 1$ and $M\tau < 1$. Using these approximations the ratio $\frac{\sigma_m^2}{m}$ reduces to

$$\frac{\sigma_m^2}{m} = 1 - 2M\tau e^{-M\tau}$$

This is also plotted in Figure 3 with the curve for non extended dead time.

Figure 3 shows that there is a difference in standard deviation if the dead time is extended or non extended. By measuring the standard deviation and the counting rate it would then be possible to calculate the

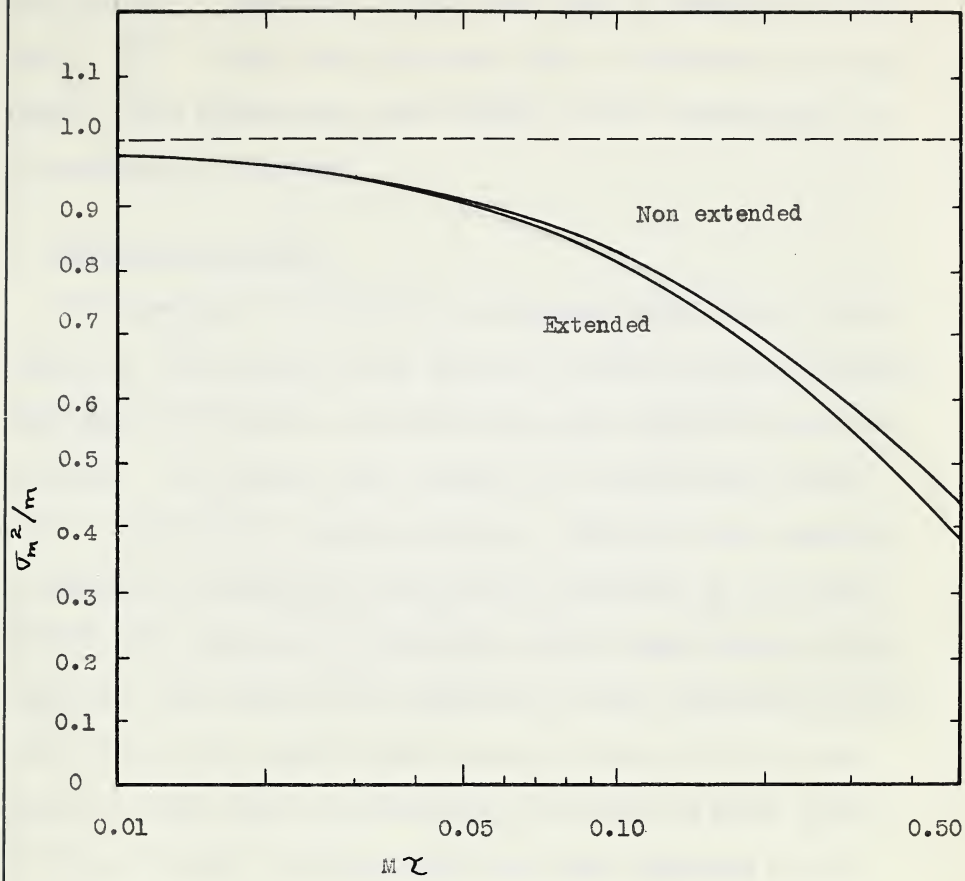


Fig. 4.

The ratio of the measured variance to the measured mean for extended and non extended dead times as a function of the product, mean pulse rate times dead time.

dead time from $\frac{\sigma_m^2}{m}$ if it is known to be either extended or non extended. Conversel , if the dead time is measured by other means, $\frac{\sigma_m^2}{m}$ will tell if the dead time is extended or non extended. This analysis may prove helpful in the investigation of the mechanism of counters.

B. Counting Rate Meter.

The counting rate meter is an instrument which gives a direct reading of the average rate at which the pulses are being received. Many types of counting rate meters have been designed (24,25,26, 27,28,29). In general, they consist of an integrating circuit with an exponentially decaying memory. For each count received, a quantity of charge, q, is fed on to a condenser of an RC tank circuit. The charge on the condenser slowly leaks through the resistance. The voltage on the condenser at which equilibrium is attained between the rate at which charge is being put on and the leakage current, gives an indication of the rate at which pulses are being received. This indicated rate will fluctuate if the pulses being received are not regular. The extent of the fluctuations will be determined by the values of R and C in the tank circuit, as well as by the frequency distribution of the pulses.

Fig. 5 shows a copy of a recording of a trace from various samples taken with the rate meter which forms the basis for the standard deviation meter.

Fig. 6 shows a calibration curve of meter deflection vs. measured

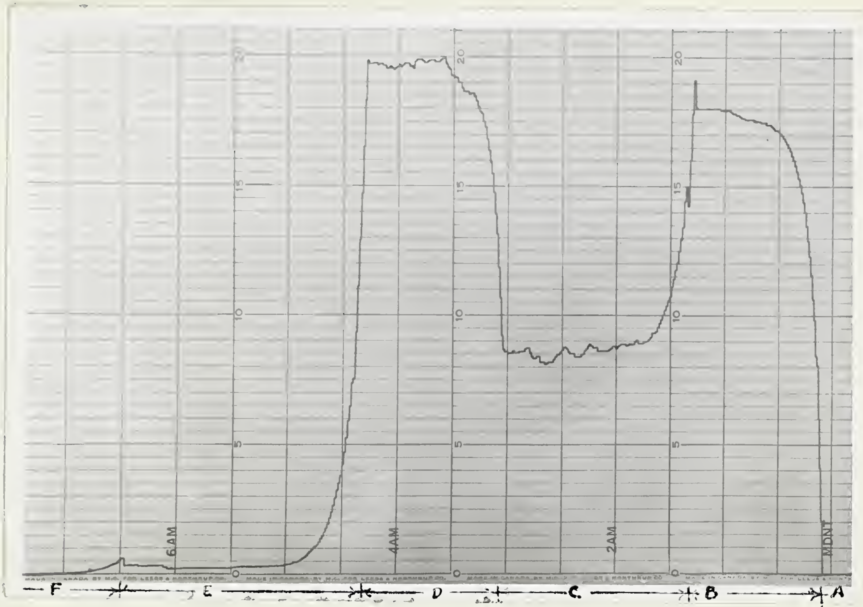


Fig. 5.

A copy of a typical counting rate meter trace.

A, Zero setting

B, 3600 c/m on 60 cycle test

C, Sample, 1780 c/m

D, Sample, 3940 c/m

E, Background, 50 c/m

F, Zero

Total running time, 30 minutes

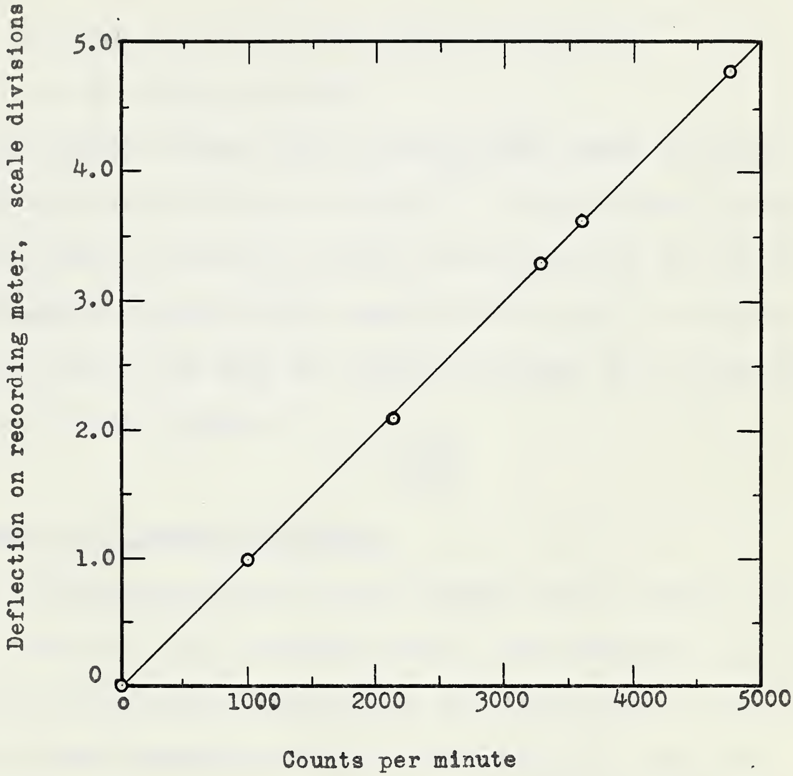


Fig. 6.

Calibration of the counting rate meter on the 5000 counts per minute range.

counting rate. The relation is seen to be linear.

An analysis of the counting rate meter by L.I.Schiff and R.D.Evans (30) has shown that the average voltage indicated after the instrument has been receiving pulses for a time long compared with RC is given by

$$\overline{V} = MqR$$

where :

M is the rate at which the pulses are received

q is the charge per pulse.

This analysis assumes that a constant mean number of pulses is received in each interval of time dt. If the frequency distribution of pulses is assumed to follow the Poisson law, the variance of the number of pulses in each interval dt is equal to the mean. It is then shown (30) that the standard deviation of an instantaneous reading of V will be given by

$$\sigma = q\sqrt{\frac{RM}{2c}}$$

<u>Advantages of a Counting Rate Meter.</u>

1. The counting rate may be read directly from a meter or recorded continuously by a recording meter. With continuous recording any change in the counting rate during the interval will be detected.

2. A rough indication of standard deviation is obtained by observing the fluctuations of the meter.

<u>Disadvantages of a C.R.M.</u>

1. The accuracy of a counting rate determination is limited by the accuracy of calibration and the stability of the components involved. Using ordinary radio components, frequent checks must be

made on a calibration standard to correct for meter drift.

Calibration of the Counting Rate Meter:

The counting rate meter was calibrated by feeding in regular pulses of a known frequency. These were obtained by tapping off a sweep frequency from the horizontal plates of an oscilloscope, synchronized to various multiples and fractions of 60 cycle line frequency. The lowest frequency available in this manner was 15 cycles per second or 900 counts per minute. This would calibrate down to the 1000 c/m scale. For frequencies below this, the output of the scaler on 60 cycle test was used. 450 c/m were obtained by using the output from the scale of 8 with 60 cycle input. 56.25 c/m were obtained from the output from the scale of 64.

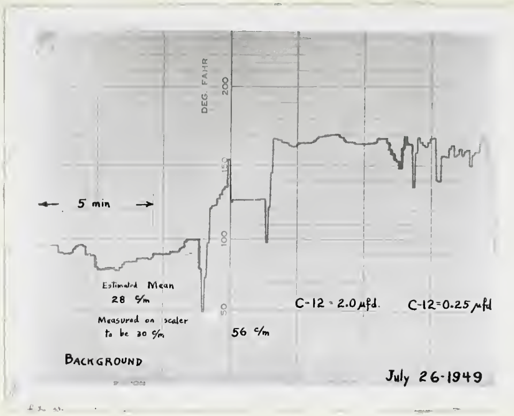
The calibration frequencies with their source are listed in Table 2.

TABLE 2.
CALIBRATING FREQUENCIES FOR THE COUNTING RATE METER.

Counts per minute	Source.	
18,000	Oscilloscope sweep -	5 traces
14,400	"	4 "
10,800	"	3 "
7,200	"	2 "
3,600	"	1 cycle
1,800	"	2 cycles
1,200	"	3 cycles
900	"	4 cycles
450	Scale of 8	
56.25	Scale of 64	

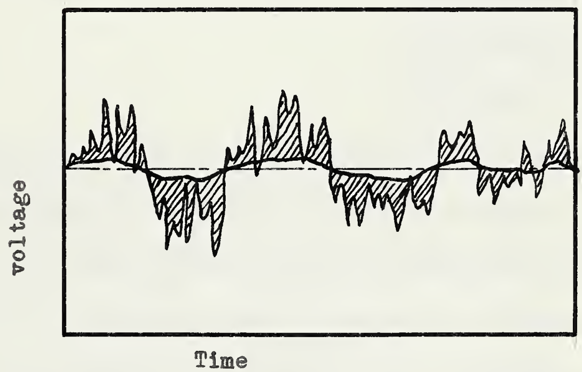
Fig. 7.

A.



Recording of the rate with different time constants

B.



Two traces as obtained from integrating circuits with different time constants. The shaded area between them is proportional to the mean deviation.

(x-M). The mean of the absolute value of this voltage difference is obtained by using a full wave rectifier and RC smoothing circuit. This is the mean deviation which has been shown to be proportional to the standard deviation for a normal distribution, and for a Poisson distribution with a large mean.

The counting rate meter chosen as a basis for the standard deviation meter was essentially the one designed by A.Kipp, A.Bousquet R.Evans, and W.Tuttle, (28), except for some additions. Its operation is in brief as follows: (where the components refer to Fig.8 Figs 9A and 9B show the layout of the principal circuit elements on the two chassis).

A quenching circuit and preamplifier is centered around V-1. V-2 gives further amplification to trigger the single-shot multivibrator formed by V-3 and V-4. This multivibrator is the pulse shaping device to give a uniform charge for each pulse. V-5 further amplifies the pulse and inverts it to feed a charge on to C-12. C-12 with R-19, R-20 and R-21, form the integrating tank circuit. C-12 is 2 microfarads and each resistance is 10 megs. This gives time constants of 20,40 and 60 seconds for the various ranges.

V-6 forms a vacuum tube voltmeter to read the voltage on C-12. This voltage is indicated by the current in M-1 or the potential across J-1. Calibration adjustments for each range are made by varying the voltage on the screen grid of V-5, which controls the size of the pulse fed on to C-12. Over-all sensitivity of the meter is controlled by R-48. Zero-ing of the meter is done by adjustment of R-47.

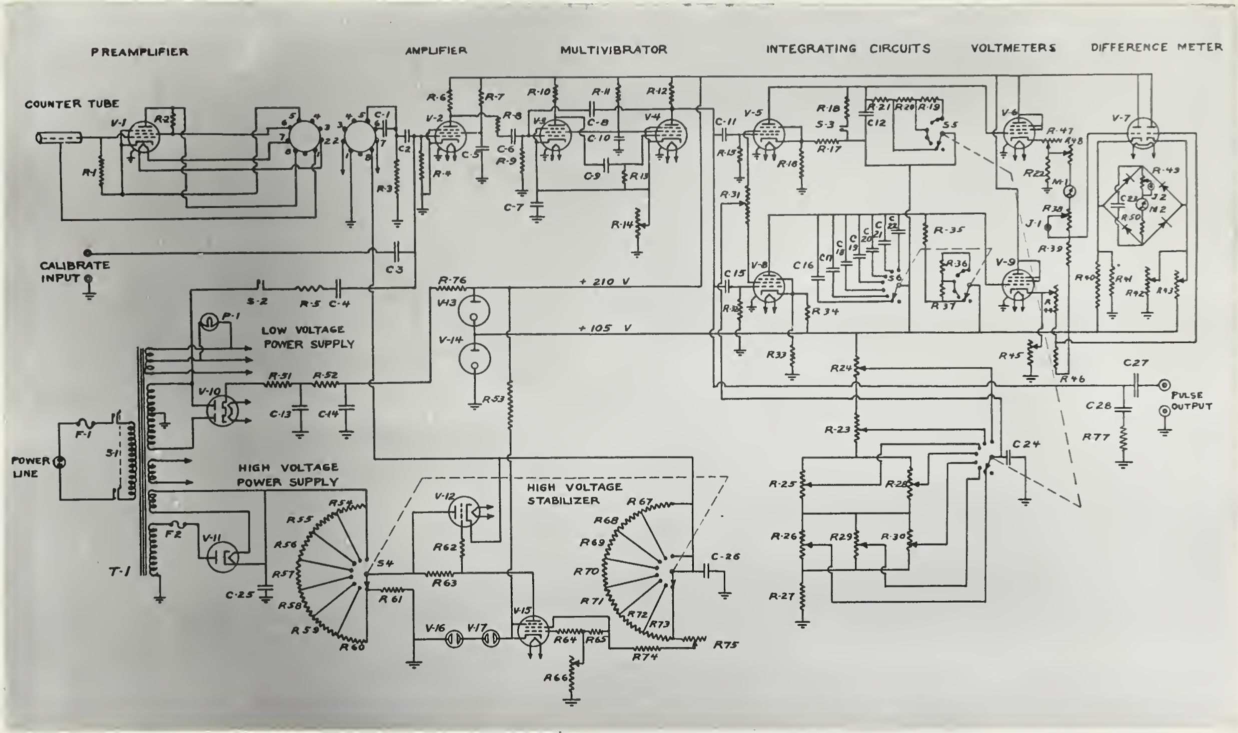


Fig. 8. Circuit diagram of the counting rate meter and standard deviation meter.

The components are listed in Table 5.

Table 5

V1	6J7
V2	6SJ7
V3	6SJ7
V4	6SJ7
V5	6SJ7
V6	6AC7/1852
V7	6C8
V8	6SJ7
V9	6AC7/1852
V10	6x5
V11	2x2/879
V12	6C5
V13	VR105
V14	VR105
V15	6C6
V16, V17	1/4 watt neon bulbs, bayonet base without internal resistors.
R1	10 meg. 1 watt.
R2	5 meg. 1 watt.
R3	8 meg.
R4	0.5 meg. 1/4 watt
R5	1 meg., 1 watt.
R6	200,000, 1 watt.
R7	300,000 1 watt.
R8	1 meg. 1/4 watt.
R9	100,000 1/2 watt.
R10	250,000 1 watt
R11	300,000 1/2 watt
R12	250,000 1 watt
R13	100,000 1/4 watt
R14	6,000 Pot.
R15	0.5 meg. 1/4 watt.
R16	3300 10 watt.
R17	30,000 1 watt
R18	100,000 1/4 watt.
R19	10 meg. 1/4 watt
R20	10 meg. 1/4 watt.
R21	10 meg. 1/4 watt.
R22	12,000 10 watt.
R23	25,000, Pot.
R24	50,000, Pot.
R25	15,000 Pot.
R26	15,000 Pot.
R27	8,500
R28	15,000 Pot.
R29	15,000 Pot.
R30	15,000 Pot.
R31	5,000 Pot.
R32	0.5 meg. 1/4 watt.
R33	3500 - 10 watts
R34	30,000 - 1 watt.
R35	10 meg. 1/4 watt.
R36	10 meg. 1/4 watt.
R37	10 meg. 1/4 watt.
R38	50 pot.
R39	2000 1/2 watt.
R40	5000 1 watt.
R41	25000 1 watt.
R42	5000 Pot.
R43	5000 Pot.
R44	5000 Pot.
R45	10,000, 10 watts variable + 500 pot.
R46	2000
R47	500 Pot.
R48	5000 Pot.
R49	5000 Pot.
R50	15,000 1/2 watt.
R51	150, 1/2 watt.
R52	1,000, 10 watts variable
R53	300,000, 2 watts
R54	0.1 meg. 1/4 watt.

(continued)

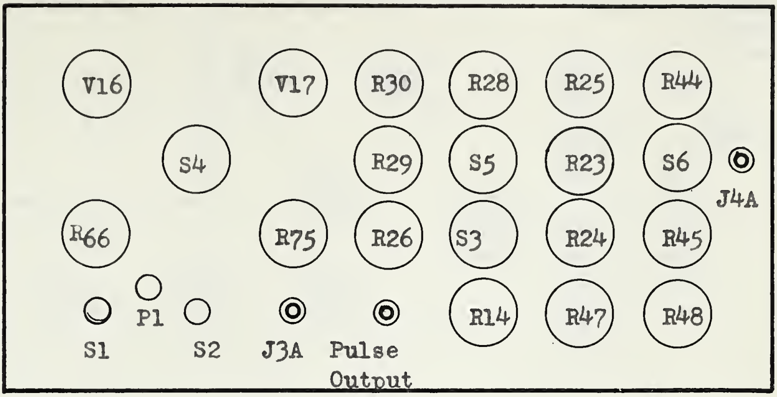
R55	0.2 meg. 1/4 watt.
R56	0.3 meg. 1/4 watt.
R57	0.3 meg. 1/4 watt.
R58	0.25 meg. 1/4 watt.
R59	0.25 meg. 1/4 watt.
R60	0.25 meg. 1/4 watt.
R61	2.25 meg. 1/4 watt.
R62	10 meg. 1 watt.
R63	100 meg. 1/4 watt.
R64	1 meg. 1/4 watt.
R65	0.5 meg. 1/4 watt.
R66	1 meg. pot.
R67	0.75 meg. 1/4 watt.
R68	0.75 meg. 1/4 watt.
R69	0.75 meg. 1/4 watt.
R70	0.75 meg. 1/4 watt
R71	0.75 meg. 1/4 watt
R72	0.75 meg. 1/4 watt
R73	0.75 meg. 1/4 watt
R74	0.5 meg. 1 watt.
R75	1 meg. Pot.
R76	3,500,10 watts variable
R77	0.5 meg.
C1	0.5 1000 volts
C2	50 ppf ceramic
C3	0.01 mica.
C4	0.002 mica.
C5	0.25 paper
C6	27 ppf
C7	0.5 400 V. paper
C8	0.0001 mica
C9	0.0001 mica
C10	0.25 paper
C11	0.0002 mica.
C12	2 paper

C13	20 450 v. electrolytic
C14	20 450 v. electrolytic
C15	0.0002 mica.
C16	0.00094 mica (cal. 1%)
C17	.00188 mica (cal. 1%)
C18	.00375 mica (cal. 1%)
C19	.0094 mica (cal. 1%)
C20	.0188 mica (cal. 1%)
C21	0.094 paper (cal. 1%)
C22	0.094 paper(cal. 1%)
C23	2000-50V Mallory 2-44
C24	0.1 paper 161,511
C25	0.25 2000V. oil
C26	0.25 2000V oil
C27	50 ppf. mica
C28	50 ppf. mica
P1	6.3V pilot light
F1	2 amp 200V fuse
F2	1/32 amp fuse
J1	Jack - output to recording Potentiometer
J2	Jack - output to recording Potentiometer
M1	0 5 or 0 2.5 ma.
M2	0 100 micro amps.
T1	RCA Transformer No. 33390 Pri. 117 Volts 60 cy. L.V. Plate 330 - 0 - 330 V 140 ma. Fila: 2.5 V., 1.75 amp. Fila. 5.0 V., 3.0 amp. Fila. 6.3 V., 0.6 amp. Fila. 6.6 V., 0.3 amp. Fila. 12.6/6.3 V, 3.82 amp

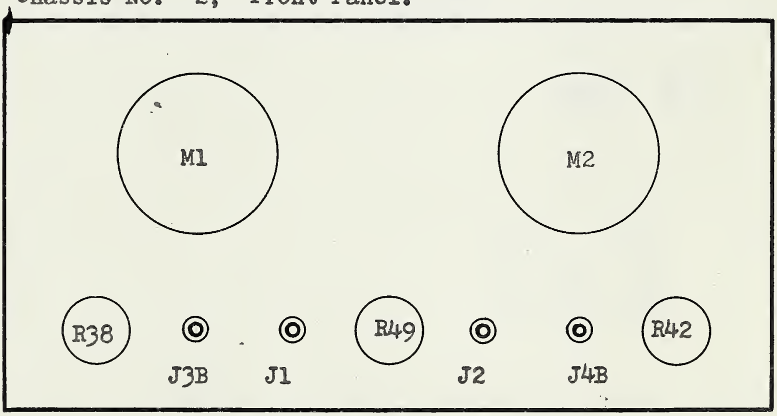
Resistances in ohms and capacities in microfarads unless otherwise noted.

Fig. 9 A. Position of Circuit Elements.

Chassis No. 1, Front Panel.



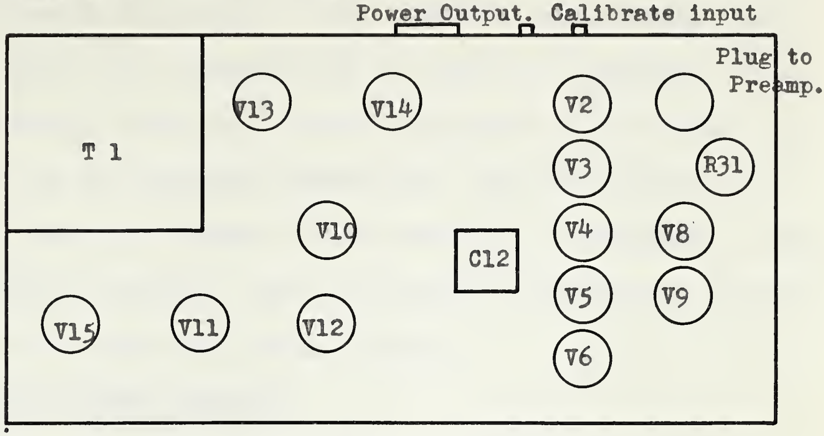
Chassis No. 2, Front Panel.



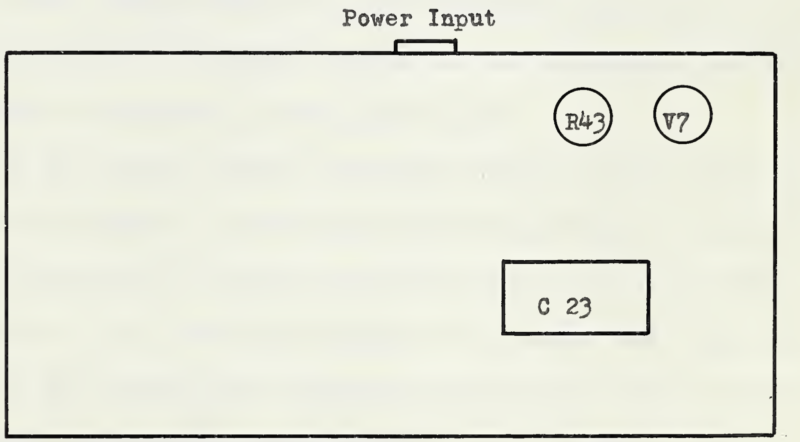
Note - J3A and J4A are connected directly to J3B and J4B.

Fig. 9 B. Position of Circuit Elements

Chassis No. 1. Top View.



Chassis No. 2. Top View.



Power output and input sockets:

Pin No. 2, +210 V

3, +105 V

4,7, 6.3 V

6, Ground

It would be possible to read $(x-M)^2$ by using a high resistance R.M.S. a.c. voltmeter such as a hot wire instrument. This, when averaged, would give a direct indication of the standard deviation for any frequency distribution. This has not been done in this instrument because of the difficulty in obtaining the long time constant desired. Also the recording potentiometer available could be used only with a d.c. voltage .

Choice of Circuit Elements:

Circuit elements for the standard deviation circuit were chosen to fulfill the following requirements.

(a) The average voltage on the integrating tank circuit should be the same as on the circuit reading M. The difference in voltage could then be measured and would give $(x-M)$.

(b) The average voltage difference between the two circuits should be as large as possible and still assure that the instantaneous rate would not exceed the average rate by an amount which would put it on a non linear region of the vacuum tube voltmeter.

(c) The counting rate indicated by M-1 should not be disturbed in reading the standard deviation because it is usually the most important quantity.

These conditions were met as follows:

(a) The average voltage on the tank condenser is given by

$$\overline{V} = M q R \qquad (30)$$

This shows that if the charge per pulse and the leakage resis-

tance in the two circuits were made the same, the average voltage would then be the same. Therefore in Fig.8, R-19, R-20, and R-21 were made equal to R-35, R-36 and R-37. The charge q was made the same by feeding the pulse from the multivibrator , to two amplifiers, one for each section. The relative sizes of the pulses were controlled by control of screen grid voltages.

(b) The average voltage difference between the circuits is given approximately by the difference in standard deviations. The standard deviation in voltage has been shown to be:

$$\sigma_V = q\sqrt{\frac{MR}{2C}}$$

q, M, and R, will be the same in the two circuits, and if we let C_1 and C_2 be the condensers in the circuits for M and x respectively, the voltage difference will be approximately:

$$V = q\sqrt{\frac{MR}{2}\left(\frac{1}{C_2} - \frac{1}{C_1}\right)}$$

If we may regard C_1 as being large compared with C_2 this becomes

$$V = q\sqrt{\frac{MR}{2C_2}}$$

The values of C_2 may be obtained by first taking the ratio

$$\frac{V}{\overline{V}} = q\sqrt{\frac{MR}{2C_2}}\left(\frac{1}{MqR}\right)$$

and then solving for C_2

$$C_2 = \left(\frac{\overline{V}}{V}\right)^2 \frac{1}{2MR}$$

For full scale deflection $\overline{V}$ was measured to be 12.5 volts. Then if we make V equalto the square root of $\overline{V}$,

$$C_2 = \frac{12.5}{2\,M\,R}$$ where M is the counting rate in counts per second for full scale deflection.

TABLE 4 shows the capacities calculated for each range.

Range in c/m	R megs	C μfarads.
20,000	10	0.00094
10,000	20	0.00188
5,000	20	0.00375
2,000	20	0.0094
1,000	30	0.0188
500	30	0.0375
200	30	0.094

The voltages at the cathodes of the tubes V-6 and V-9 also give an indication of M and x. If a voltmeter was put between these points the current required to run it would come through M-1 and therefore disturb the counting rate reading.

This was prevented by using a vacuum tube voltmeter drawing no current. The two voltages were put on to the grids of the double triode V-7. These voltages were varying from 105 to about 95 volts. The cathode bias resistors were chosen to make the grids about 1 volt negative with no current in R-44. The voltages at the cathodes would then indicate M and x.

The difference in cathode voltage was indicated by M-2. A full wave copper oxide rectifier made it indicate the absolute difference. The condenser C-23 across the voltmeter was chosen to give a time constant of approximately 30 seconds. The voltage difference was often about 1.5 volts. Using a 100 micro amp meter this

would require a meter resistance of 15,000 ohms. Then to give a time constant of about 30 seconds, C must be 2,000 μfarads. This is ordinarily a prohibitively large capacity, but the voltage is low and such large capacities are available at low voltage.

Adjustments for the Standard Deviation Meter:

(a) Zero Setting for Zero Input.

The rate meter, M-1, Fig 8 was first adjusted to zero with R-47. The voltage on the left grid of V-7 was then 105 volts. R-40 was chosen to make the cathode relatively negative by about 1 volt at this adjustment. R-45 was then adjusted to give an equal potential on the other grid of V-7. This was obtained when there was no current in R-44, and hence adjustment of R-44 produced no change on the grid and hence no charge in the reading indicated by M-2.

R-42 was then adjusted to make the cathode potentials of V-7 equal and no current in M-2.

(b) Zero Setting for a Regular Input (σ=0).

A regular pulse such as 60 cycles was fed in to the meter and the meter allowed to come to equilibrium. The size of the pulses fed on to the tank circuits were made equal by adjusting R-31 until a minimum was obtained in M-2 with M-1 indicating the correct counting rage. The counting rate was adjusted by the appropriate resistance in the network, R-24 to R-30 and R-48.

Finally a minimum was obtained in M-2 by adjustment of R-43.

A true zero could not be obtained because of the nature of the meter.

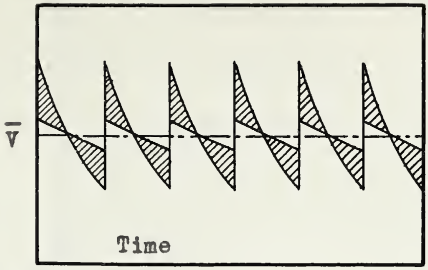


Fig. 10.
Voltages with regular input

Fig.10 shows an exaggerated plot of the voltages on the tank condensers of the circuits with the two time constants. The average absolute difference in voltage is indicated by the shaded portion. This was not found to be large but does put a limit on the accuracy of the instrument. See Fig. 11

Calibration of the Standard Deviation Scale:

An absolute calibration may be made only by comparing the reading of M-2 with the standard deviation as obtained by direct calculation.

To do this the pulses from the pulse output terminals were fed into the scaler. A series of n scaler readings were taken while the potentiometer was recording the deflection indicated by M-2. The Standard Deviation was calculated from the readings by using the procedure outlined previously.

A graph was then plotted to correlate the average meter deflection with the actual standard deviation. This graph, Fig.14, also shows the standard error of the calculated σ.

Linearity:

To test for linearity of the scale it was necessary to calibrate several points. To obtain 1% error would have been very laborious so a compromise of approximately 100 readings per point

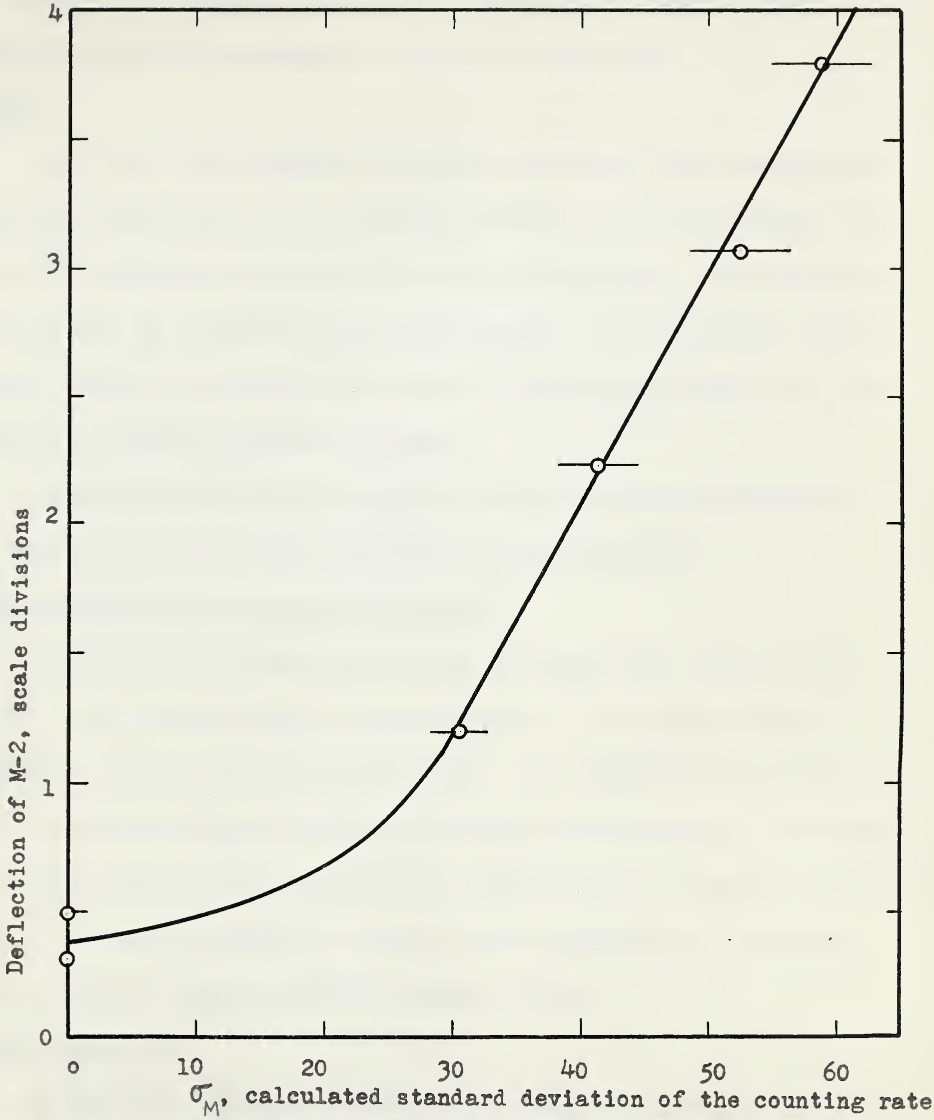


Fig. 14. Calibration curve for the standard deviation meter on the 5000 c/m scale. One scale division equals 10 microamps.

were taken, which gave a standard error of approximately 7%. At 25 seconds per reading, it was then necessary to count for approximately three-quarters of an hour on each point.

Drift:

Over the time required for several readings there occasionally appeared some drift in the recorded reading. It was necessary when this was noticed on the trace to check on 60 cycles. The difficulty was usually in the pulse splitting elements. Adjustment of R-31 to give a minimum in m-2 gave the correct counting rate again and the original standard deviation reading.

For this reason it is suggested that any further instruments of this type be made with two pulse shaping circuits.

Experimental Procedure for Calibration:

An externally quenched RCL Model 20 Geiger tube was used to detect the radiation from a radium sample. The pulses were put directly into the counting rate meter. The output of the meter was put into the Scaler and the recording potentiometer. The counting rate, from J-1 was recorded for approximately 10 minutes, and then the standard deviation from J-2 for approximately 10 minutes. Fig. 11 shows a ~~photograph of a~~ typical trace.

TIMING CIRCUIT.

A series of 15 second readings were taken on the scaler. The timing was done using an automatic control circuit as shown in Fig. 12. Basically it is an Eccles Jordan Trigger circuit with a relay in one arm. The relay closes the "count" switch on the scaler

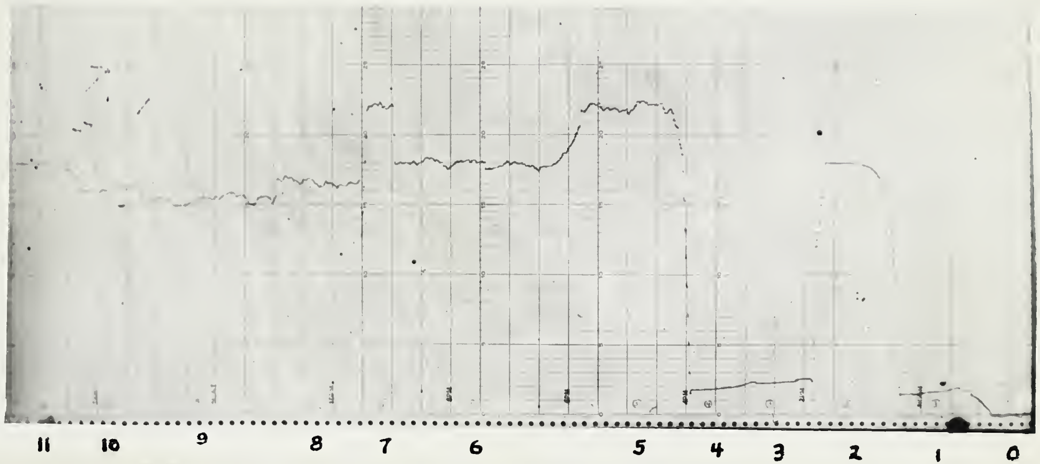


Fig. 11, Counting Rates and Standard Deviations

0 - Zero on C.R.M.

1 - Zero on o meter

2 - 60 cycles on C.R.M.

3 - o, 60 cycles

4 - Zero on o meter

5 - Sample No. 1

6 - o , sample No. 1

7 - Check on sample No. 1

8 - Sample No. 2

9 - o , sample No. 2

10- Check on No. 2

11- Check on 60 cycles

when that arm is conducting. The pulses to actuate it were obtained from an electric clock with two fixed contacts and a "cat's whisker" placed on the sweep second hand. A 25 cycle electric clock was used, and run on 60 cycle current. Under this condition the sweep second hand completed one revolution in 25 seconds. The contacts were adjusted to turn the meter on for 15 seconds and then off for 10 seconds. Ten seconds was found ample to read the scaler and register and reset for the next reading. The scaler was an Atomic Instrument Co. Model 101A, scale of 64. The output of this scale was put into a Cyclotron Specialties Co. Mechanical register.

The relay was later put into the counting rate meter output as illustrated in figure 13. This led to no spurious pulses introduced into the scaler. In this case also, the contacts on the clock remained closed for the 15 seconds during which counting was carried out. Having the relay so close to the pulse splitting network meant that the size of the pulses were affected by closing and opening the relay. It was then necessary to do the counting while the recorder was off.

The timing interval was calibrated by counting a known regular frequency. Fig. 15 shows the arrangement of the apparatus in the ~~lab.~~ laboratory. On the left is the oscilloscope from which the regular calibration frequencies were obtained. Next to it is the counting rate and standard deviation meter. Then the scalers and the recording meter on the extreme right. The timing circuit is shown in front of the counting rate meter.

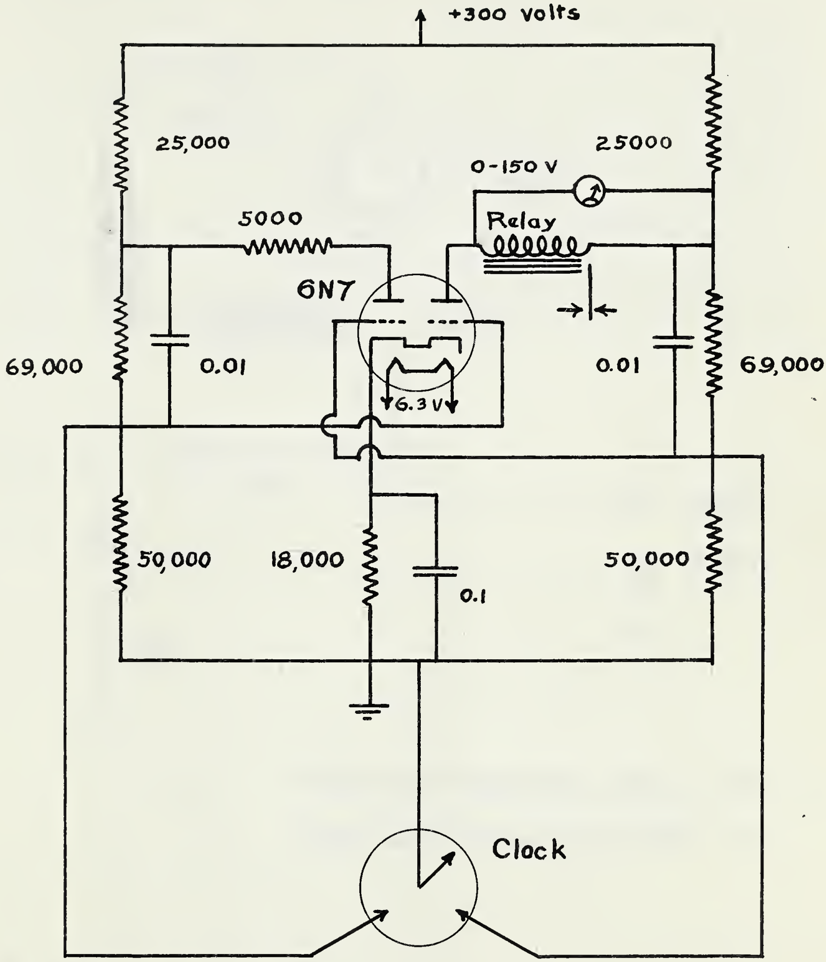


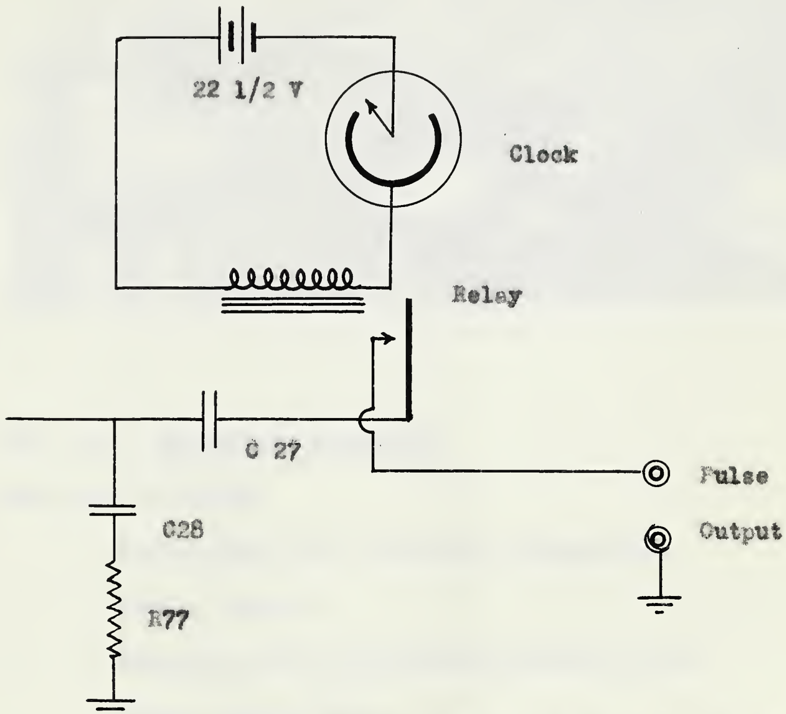
Fig. 12.
Timing circuit.

Resistances in ohms

Capacities in microfarads

Relay - Struthers Dunn Inc. Type 28XAX122
Coil current, D.C. 3 m.a.

Fig. 13. Timing Circuit



Numbers of Components refer to Fig. 8.

Relay and Clock are as in Fig. 12

Fig. 15. Laboratory Apparatus

From left to right,

Oscilloscope for calibration frequencies

Timing network

Counting rate and standard deviation meter

Scalar and register

Voltrol to control speed of recording paper

Recording potentiometer

Measurement of the time interval

The time interval was measured by making a series of counts of pulses obtained from 60 cycles line frequency. In the notation already described, x is the number of counts recorded per interval. To simplify calculations a computation variable y was chosen, such that the deviations of x from y were small.

In the series of 20 readings which were made the average was about 840 counts. Therefore y was chosen to be 840 - x. The mean number of counts per interval was then $\bar{y} + 840$, and the standard deviation, $\overline{y^2} - \bar{y}^2$

A summary of the calculations follows:

$$n = 20$$

$$y = 5.$$

$$\bar{y} = \frac{5}{20} = 0.25$$

$$\therefore m = 840.25.$$

The pulses were occurring at the rate of 3600 per minute so the average time interval was $\frac{840.25}{3600} = 0.23335$ minutes.

Each value of y was squared, and the sum is

$$\sum (y^2) = 41$$

$$\text{So} \quad \overline{y^2} = \frac{41}{20} = 2.05.$$

$$\sigma_m^{\ 2} = \overline{y^2} - \bar{y}^2 = 1.42$$

$$\sigma_m = \sqrt{1.988} = 1.41$$

The standard deviation in the time interval was then $\frac{1.19}{840.1} \times 0.23335$ minutes = 0.0004 minutes.

The time interval was then taken to be 0.2334 minutes with a standard deviation of 0.0004 minutes.

<u>Calculation of the standard deviation for sample Number 2</u>

$n = 100$

$T = 0.2334$ minutes

$\sigma_T = 0.0004$ minutes

$y = x - 500$

$\bar{y} = -5.45$

$\bar{y}^2 = 29.70$

$\overline{y^2} = 430.57$

$m = y + 500 = 494.55$

$\sigma_m^{\;2} = (\overline{y^2} - \bar{y}^2) = 400.87$

$\sigma_m = 20.0$

$\sqrt{m} = 22.24$

The standard error of σ_m is $\dfrac{\sigma_m}{\sqrt{200}} = 1.41$

The square root of the mean is within twice the standard error of the measured standard deviation, and the distribution has therefore not been shown to differ significantly from a Poisson distribution.

The standard deviation of the counting rate, as defined in appendix 1, is given by

$$\sigma_M = \frac{20.0}{\sqrt{0.2334}} = 41.4$$

The standard error of σ_M is $\dfrac{1.41}{\sqrt{0.2334}} = 3.0$

Consideration of timing errors

The observed variance of the counts received will be the sum of the variances due to timing errors, and the frequency distribution of the pulses. We desire only the variance of the frequency distribution.

The standard deviation in m due to the timing error will be

$$2119 \times 0.0004 \text{ counts} = 0.85 \text{ counts.}$$

The variance is then $0.85^2 = 0.72$

The measured variance was 400.87.

The variance because of the frequency distribution of the pulses is then $400.87 - 0.72 = 400.2$, and the desired standard deviation, $\sigma_m = \sqrt{400.2} = 20.0$.

The standard deviation is not changed significantly when the timing error is considered. Therefore this correction has been neglected in calculations of the standard deviations for the points on the calibration curve, Fig. 14, p 33.

Table 6

Summary of Data for the calibration curve for the standard deviation meter on the 5000 c/m scale.

Source	c/m scaler	σ_M, scaler	s.e. of σ_M	σ meter scale division
Zero	0	0	0	0.3
60 cycles	3600	0	0	0.45
Sample No. 1	3276	52.5	3.8	3.07
Sample No. 2	2119	41.4	3.0	2.22
Sample No. 3	1003	30.9	2.2	1.20
Sample No. 4	4848	59.4	4.3	3.8

Sample no. 4 was measured, 24 hours after the others. Its location in Figure 14 in the same line as the other points gives an indication of the reproducibility of the results.

Multiple Counts;

Measurements were also made using multiple pulses obtained by adjusting R-14 until the multivibrator was slightly unstable and would give 2 or more pulses for each one entering it. Fig.16 shows a trace for which two counting rates from the same input show the same standard deviation but different rates because of multiple counting

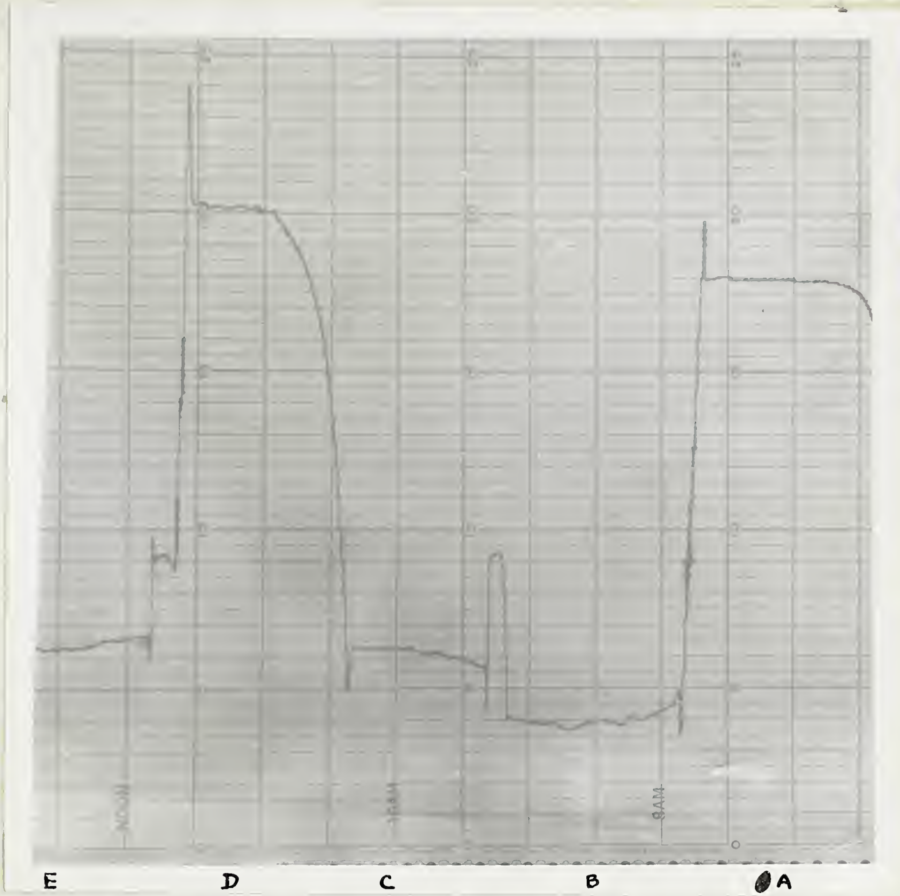


Fig. 16. Recording to show that the standard deviation reading does not depend significantly on the counting rate.

A - 60 cycle test

B - Sample , 800 c/m

C - o for sample in B. 32 c/m.

D - Same sample as in B with 5 counts registered for each one entering the counter.

E - o for D, which is the same as the o for B.

<u>Results</u>:

Figure 14 illustrates that there is a correlation between the computed standard deviation of the counting rate of a sample, and the deflection of the meter M2. Fig. 16, shows that the meter deflection is not significantly dependant on the counting rate, at least if the meter deflection is not small. In Fig. 16 the standard deviations of two samples showing different counting rates, but with theoretically the same standard deviation, do show the same average deflection on the standard deviation meter to within about 2% after equilibrium is reached To obtain a similar comparison to the same accuracy by computing the standard deviation from a series of samples would require one thousand readings (Fig. 1) of each sample.

The accuracy of the standard deviation meter is limited by the zero reading as illustrated in Fig. 10, which is expected to be a function of the counting rate. This zero correction is apprecialbe only for small meter deflections for which it would then be a significant fraction of the indicated reading. If the distribution dealt with is approximately Poissonian . There is usually no need to work with small scale deflections because a lower range of the meter may be chosen.

The difficulty in making an accurate calibration of the standard deviation scale also sets a limit on its accuracy. As illustrated in Fig. 16, the meter may be used to give comparative standard deviations, and hence detect some of the counter defects outlined at the beginning of the thesis.

Theoretical calibration, either by the measurement of voltages,

or by measuring ~~of~~ the standard deviation of a sample which may reasonably be expected to follow the Poisson distribution within the accuracy of the instrument are suggested as being expected to give a more reliable calibration. Such a sample could be obtained from a scintillation counter, counting at a rate low enough so that the effect due to dead time of the counter tube and circuits would not be significant. The standard deviation would then be very close to the square root of the mean.

Acknowledgements

This work was made possible by a grant from the Carnegie Research Fund at the University of Alberta. Thanks are also extended to Dr. W. W. Happ for suggesting the topic, and for his constant support and extensive aid in carrying out the work. I have appreciated, too, the kind assistance of Professor E. S. Keeping and Dr. D. B. Scott in dealing with the numerous mathematical problems that have arisen in the course of the work.

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Standard Deviation Meter.

The principle of the meter is that it reads the two quantities, average counting rate over a long period, and the counting rate over a short period. This is accomplished by using two integrating circuits with different time constants or 'memory' times. The long time constant circuit gives an indication of the average rate, M, and the short time constant circuit gives an indication of the actual rate x over a short period.

Fig. 7 shows how the voltage may vary on each circuit. The shaded area between the two curves gives an indication of the deviation of the actual counting rate from the mean.

The time constants used for the various ranges are listed in Table 3.

TABLE 3.
THE TIME CONSTANTS IN THE VARIOUS RANGES OF THE METER.

Range	CR for the mean	CR for the actual rate	Ratio.
20,000 c/m	20 sec.	0.0094 sec.	2120
10,000	40	0.0375	1060
5,000	40	0.075	540
2,000	40	0.188	212
1,000	60	0.563	106
500	60	1.13	54
200	60	2.82	22

Both integrating circuits are made to read the same average voltage so that the instantaneous voltage difference gives a reading

Appendix 1

Definition of σ_M as used in this thesis

The standard deviation of the counting rate, σ_M, as indicated by the standard deviation meter is to be the standard deviation which would be obtained if the readings were taken for unit time, in this case for one minute. This can be done because with the decaying memory of the instrument any arbitrary time interval may be chosen as a unit for calibration.

In calibrating the instrument, if readings are taken for a time interval T, the relative variance, σ_m^2 / m, would be expected to be the same as would be obtained if the readings were taken for unit time.

That is, $$\frac{\sigma_m^2}{m} = \frac{\sigma_M^2}{M}$$

and since $M = \frac{m}{T}$, this reduces to

$$\sigma_M = \frac{\sigma_m}{\sqrt{T}} .$$

The standard deviation of an estimate of the counting rate obtained by counting for an interval T, is given by σ_m / T. This may be obtained by taking $\sigma_M / \sqrt{T}$. It will be noted that this is the same as if T readings were taken for the unit interval. The standard deviation is reduced by the square root of the number of readings, in this case T.

With σ_M as defined above, the standard deviation meter may be used obtain an estimated standard deviation of the rate after counting for a time interval which is known. During the

counting, however, it gives the standard deviation of the rate as though a unit time interval was being used. Then the indicated σ_M should on the average be equal to the square root of the indicated rate if the pulses counted are following the Poisson distribution.

An analysis of the expected standard deviation of the indicated σ_M has not yet been carried out.

Appendix 2.

Mean deviation of the Poisson distribution.

The following analysis was presented by Mr. E. S. Keeping, and also worked out independently by Dr. W. W. Happ.

Let m = a parameter of the distribution

$[m]$ = the integer equal to or next below m

The mean deviation is:

$$\text{m.d.} = \sum_{x=0}^{[m]} (m-x)P_m(x) + \sum_{[m]+1}^{\infty} (x-m)P_m(x)$$

$$\text{where} \quad P_m(x) = \frac{e^{-m} m^x}{x!}$$

$$\text{and} \quad \sum_0^{\infty} P_m(x) = 1$$

$$\text{Therefore,} \quad \sum_{[m]+1}^{\infty} P_m(x) = 1 - \sum_0^{[m]} P_m(x)$$

$$\text{from which} \quad \sum_0^{[m]} m\,P_m(x) - \sum_{[m]+1}^{\infty} m\,P_m(x) = -m + 2m \sum_0^{[m]} P_m(x)$$

$$\text{and} \quad \sum_0^{[m]} x\,P_m(x) = \sum_1^{[m]} x\,P_m(x)$$

$$= m \sum_1^{[m]} \frac{e^{-m} m^{x-1}}{(x-1)!}$$

$$= m \sum_0^{[m-1]} \frac{e^{-m} m^x}{x!}$$

$$= m \sum_0^{[m]-1} P_m(x)$$

$$\sum_{[m]+1}^{\infty} x\,P_m(x) = m\sum_{[m]+1}^{\infty} \frac{e^{-m}\,m^{x-1}}{(x-1)!} = m\sum_{[m]}^{\infty} \frac{e^{-m}\,m^{x}}{x!}$$

$$= m\sum_{[m]}^{\infty} P_m(x)$$

$$= m\left\{1-\sum_{0}^{[m]-1} P_m(x)\right\}$$

Therefore, $\sum_{0}^{[m]} -x\,P_m(x) + \sum_{[m]+1}^{\infty} x\,P_m(x) = m - 2m\sum_{0}^{[m]-1} P_m(x)$

Then,

$$\text{m.d.} = 2m\sum_{0}^{[m]} P_m(x) - 2m\sum_{0}^{[m]-1} P_m(x)$$

$$= 2m\,P_m([m])$$